



III Semester M.Sc. Examination, January 2018
(CBCS Scheme)
MATHEMATICS
M302T : Mathematical Methods

Time : 3 Hours

Max. Marks : 70

Instructions : 1) Answer **any five full** questions.
2) **All** questions carry **equal** marks.

Answer **any five full** questions.

1. a) Convert the following Initial Value problem into an integral equation
 $y'' + y = x, y(0) = 0, y(1) = 0.$ 7
- b) Solve the following integral equation using the method of degenerate kernel
 $u(x) = \lambda \int_0^{2\pi} \cos(x+t)u(t)dt.$ 7
2. a) Solve $g(x) = e^x - \int_0^x e^{x-t}g(t)dt$ by Laplace transform method. 7
- b) Find the Neumann series solution and resolvent Kernel for the integral
 $g(x) = (1-x) + \int_0^x (x-t)g(t)dt.$ 7
3. a) Determine an asymptotic expansion for the integral of the form
 $I(x) = \int_0^x t^{-\frac{1}{2}}e^{-t}dt$ as $x \rightarrow \infty.$ 7
- b) Use Laplace method to obtain the asymptotic expansion of
 $\int_0^{\frac{\pi}{2}} e^{-x \tan t} dt$ as $x \rightarrow \infty.$ 7



4. a) Evaluate the following using Watson's lemma $I(x) = \int_0^5 \frac{e^{-xt}}{1+t^2} dt$. 7

b) Find the leading order term of $I(x) = \int_0^\infty \cos(xt^2 - t) dt$ using stationary phase method. 7

5. a) Solve $\frac{dy}{dx} = x + y^2$; $y(0) = 1$ by Runge-Kutta method of four-slopes. Obtain $y(0.1)$ by taking $\Delta x = 0.05$. 7

b) Solve by classical Runge-Kutta explicit method of two-slopes :

$$\frac{dy}{dx} = x - y; y(0) = 0,$$

$$\frac{dz}{dx} = y - x; z(0) = 1,$$

Choose $\Delta x = 0.05$ and obtain $y(0.05)$. 7

6. Derive any multi-step (Predictor-Corrector) method of Adam to find a numerical solution of $\frac{dy}{dx} = f(x, y(x)); y(x_0) = y_0$. 14

7. a) Given the IBVP

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; 0 \leq x \leq 1; t \geq 0,$$

Subject to $u(x, 0) = x(1 - x), 0 \leq x \leq 1,$

$$u(0, t) = 0,$$

$$u(1, t) = 0, t \geq 0,$$

Obtain the solution by Schmidt finite-difference method.

Choose $\Delta x = \frac{1}{4}$ and $\Delta t = \frac{1}{64}$ and find $u\left(\frac{1}{4}, \frac{1}{64}\right), u\left(\frac{1}{2}, \frac{1}{64}\right)$ and $u\left(\frac{3}{4}, \frac{1}{64}\right)$. 7



- b) Show that the Schmidt finite difference method is conditionally stable. 7
8. a) Using the explicit finite-difference method find an approximate solution of the one-dimensional wave equation 7

$$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}; 0 \leq x \leq 1; t \geq 0,$$

subject to $u(x, 0) = x(1 - x)$

$$\frac{\partial u}{\partial t}(x, 0) = 0, 0 \leq x \leq 1$$

$$u(0, t) = 0,$$

$$u(1, t) = 0, t \geq 0.$$

Choose $\Delta x = \frac{1}{4}$ and $\Delta t = \frac{1}{64}$. Obtain the solution at first-time level.

- b) Derive the finite difference approximation for second order spatial derivative in the case of a non-rectangular region. 7
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